On Frequency-Hopping and Non-Coherent Reception in PLC Applications

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1 Introduction

A problem in power line communications (PLC) is the time-varying characteristics of the channel and the noise. Robust communication methods are therefore considered as possible candidates for PLC, see e.g. references in [6]. At the receiver, robust detection techniques are of interest, and one important issue is the amount of knowledge about the channel that is needed in the receiver. Non-coherent reception is an example of a robust technique, where the channel's influence on the phase of the carrier-wave is ignored. This simplifies the receiver since the phase of the carrier does not need to be estimated. In this paper we investigate non-coherent detection of FSK signals (orthogonal frequency shift keying) with unequal energies. Such signals are often the result of a channel having different attenuation at different frequencies. Here we focus on a minimum Euclidean distance based non-coherent receiver, which is assumed to know (through measurements) the envelope, but not the phase, of the received signal alternatives. An alternative approach is to always use a non-coherent receiver designed for the equal-energy case. However, if such a receiver is used in a situation when the received signal alternatives have different energies, the bit error probability can be significantly increased (compared with a non-coherent receiver which uses the fact that the signal energies are different). In this paper we also discuss coding combined with frequency-hopping, and non-coherent detection of OFDM-type signals (orthogonal frequency division multiplex), which is a more bandwidth efficient signaling scheme.

2 Non-Coherent Reception: Unequal Energies

In this section non-coherent detection of uncoded equally likely $M$-ary FSK signals in AWGN is studied. Hence, it is assumed that the additive noise $N(t)$ is a zero-mean white Gaussian random process having power spectral density $R_N(f) = N_0/2$. It should be noted that this is a simplified model of the noise characteristics on the low-voltage grid, see references in [6]. However, we use the AWGN model in a first attempt to obtain a robust receiver structure. If the results are promising, then it is motivated to do further studies with more realistic models. Given that message $m_j$ is transmitted, $j = 0, 1, \ldots, (M - 1)$, it is assumed that the received signal $r(t)$ is

$$r(t) = z_j(t) + N(t) = g(t)\sqrt{2E_j} \cos(2\pi f_j t + \varphi_j) + N(t)$$  \hspace{1cm} (1)

in the symbol interval $0 \leq t \leq T_s$. $E_j$ denotes the energy in the received signal alternative $z_j(t)$, and without loss of generality it can be assumed that $E_j = \alpha_j E$, where $\alpha_0 \leq \alpha_1 \leq \ldots \leq \alpha_{M-1} = 1$. Hence, the signal alternative $z_{M-1}(t)$ has the largest energy ($= E$). The symbol time $T_s$, alternatively the signaling interval, is $T_s = k/R_b$ where $k = \log_2(M)$ and $R_b$ is the information bit rate in bits per second.
In (1), \( g(t) \) represents a baseband pulse which might be used for the purpose of spectral shaping. For convenience, it is here assumed that the energy in \( g(t) \) is unity, i.e. \( \int_{0}^{T_s} g^2(t) dt = 1 \). Furthermore, \( g(t) \) and the \( M \) energies \( E_l \) are assumed to be known by the receiver, but the phase value \( \varphi_j \) in (1) is considered to be unknown. Therefore, \( \varphi_j \) is modelled as a random variable having a uniform probability density function over \([0, 2\pi]\). The non-coherent receiver considered in this paper is shown in Figure 1 below. As usual, the first step in the receiver is to extract the set of coordinates \( \{r_c,i, r_s,i\}_{i=0}^{M-1} \) in the \( 2M \)-dimensional signal space. This is in Figure 1 implemented as a bank of \( 2M \) sampled matched filters. The impulse response of the filters are matched to the orthonormal basis functions \( \phi_{c,i}(t) = g(t)\sqrt{2}\cos(2\pi f_i t) \) and \( \phi_{s,i}(t) = -g(t)\sqrt{2}\sin(2\pi f_i t) \) in the time interval \( 0 \leq t \leq T_s \) (and equal to zero otherwise). It is here assumed that the pulse \( g(t) \) and the set of frequencies \( \{f_i\}_{i=0}^{M-1} \) are properly chosen, i.e. such that the \( 2M \) functions \( \{\phi_{c,i}(t), \phi_{s,i}(t)\}_{i=0}^{M-1} \) are orthonormal (with frequency separation \( f_A \) at least \( 1/T_s \), since non-coherent detection is used).

If it is assumed that the received signal is \( r(t) = z_j(t) + N(t) \), then the coordinates obtained in the receiver are \( r_{c,j} = \sqrt{E_j}\cos(\varphi_j) + w_{c,j}, \quad r_{s,j} = \sqrt{E_j}\sin(\varphi_j) + w_{s,j} \), and \( r_{c,l} = w_{c,l}, \quad r_{s,l} = w_{s,l} \) for all \( j \neq l \). The \( 2M \) random variables \( \{w_{c,i}, w_{s,i}\}_{i=0}^{M-1} \) are due to the noise \( N(t) \), and they are independent and Gaussian, each having zero mean and variance \( \sigma^2 = N_0/2 \). So, in the noiseless case, the received \( 2M \)-dimensional signal-vector \( r \) lies on a circle with radius \( \sqrt{E_j} \). In the presence of noise we can use the Euclidean distance concept to make a non-coherent decision of the transmitted message. The receiver in Figure 1 decides that message \( \hat{m} = m_i \) was sent if and only if the the received \( 2M \)-dimensional signal-vector \( r \) is closer to the circle associated with \( z_i(t) \) (message \( m_i \)) than to any of the other \( M-1 \) circles in the signal space. This decision rule is quite robust, and can also be used in situations when the noise is non-Gaussian. It is seen in Figure 1 that the decision variables \( \{\xi_l\}_{l=0}^{M-1} \) can be expressed as \( \xi_l = R_l\sqrt{E_l} - E_l/2 \), where \( R_l = \sqrt{r_{c,l}^2 + r_{s,l}^2} \). By using well-known results for Rayleigh and Ricean random variables, see refs. [1]-[5], the symbol error probability for the receiver in Figure 1 can be evaluated. However, in this paper we restrict ourselves to the special case when \( M = 2 \). In many applications it is desired to calculate the symbol error probability as a function of the received signal-to-noise ratio \( \xi_b/N_0 \), where \( \xi_b \) denotes the received
energy per information bit on the average. This is easily done by replacing the maximum energy \( E \) with \( E = \frac{M \log_2(M)}{1 + \sum_{n=0}^{\infty} \alpha_n} \).

2.1 The Binary Case

Here we study the bit error probability \( P_b \) for the special case when \( M = 2 \) in Figure 1. In this case, \( P_b = (P_F + P_M)/2 \), where \( P_F \) and \( P_M \) denote the probability of a "false alarm" and of a "miss", respectively [3]. From Figure 1 we deduce that \( P_F \) can be expressed as \( P_F = \text{Prob}\{R_1 > \frac{\sqrt{E_0} + (E_1 - E_0)}{\sqrt{E_1}}\} \), where \( R_1 \) is Rayleigh distributed and \( R_0 \) is Ricean distributed. Hence, \( P_F \) can be evaluated according to the expression below,

\[
P_F = \int_0^\infty e^{-(y\sqrt{\alpha_0} + (1-\alpha_0)/2)\frac{E}{N_0}} 2y e^{-(y^2 + \alpha_0)\frac{E}{N_0}} I_0(2y\sqrt{\alpha_0}) dy
\]

where \( I_0(z) \) denotes the 0th-order modified Bessel function of the first kind. Furthermore, if it is assumed that \( 0 < \alpha_0 \leq 1 \), then the probability of a "miss" can be evaluated as,

\[
P_M = \int_0^{(1-\alpha_0)/2} 2y e^{-(y^2 + 1)\frac{E}{N_0}} I_0(2y\frac{E}{N_0}) dy + \int_{(1-\alpha_0)/2}^\infty e^{-(y\sqrt{\alpha_0} + (1-\alpha_0)/2)\frac{E}{N_0}} 2y e^{-(y^2 + \alpha_0)\frac{E}{N_0}} I_0(2y\sqrt{\alpha_0}) dy
\]

Observe that \( \alpha_0 \) must be positive in this expression. In the special case when \( \alpha_0 = 0 \) (on-off signals), the receiver in Figure 1 base its decision on the sign of the decision variable \( \xi_1 \), and the expressions for \( P_F \) and \( P_M \) are then considerably simplified. It is straightforward to show that the bit error probability for the receiver in Figure 1 in this special case can be written as,

\[
P_b = (e^{-\frac{E_b}{2N_0}} + 1 - Q_1(\sqrt{\frac{E_0}{N_0}}, \sqrt{\frac{E_b}{N_0}}))/2 < e^{-\frac{E_b}{2N_0}}
\]

where \( Q_1(a, b) \) denotes the Marcum's Q function, see refs. [1]-[5]. The upper bound for \( P_b \) is here obtained by observing that \( P_F = e^{-\frac{E_b}{2N_0}} > P_M \). Another special case of interest is the situation when the two received signal alternatives have equal energy, i.e. \( E_0 = E_1 \). In this case, the receiver in Figure 1 is identical to envelope- (or square-law) detection. So, for equal-energy signal alternatives, the receiver in Figure 1 is identical with the non-coherent maximum likelihood (ML) receiver. The expressions for \( P_F \) and \( P_M \) are equal in this case, and it is easy to obtain the bit error probability as \( P_b = e^{-\frac{E_b}{2N_0}}/2 \), which is the standard result for non-coherent ML reception of two equal-energy orthogonal signals in AWGN (see refs. [1]-[3]).

Figure 2 shows the bit error probability for the receiver in Figure 1 for five cases of binary FSK: \( \alpha_0 = 0 \) (on-off), \( \alpha_0 = 1/4 \), \( \alpha_0 = 1/2 \), \( \alpha_0 = 3/4 \) and \( \alpha_0 = 1 \) (equal energy), respectively. For comparison, the bit error probability obtained with coherent ML-reception of orthogonal signals in AWGN is also shown (dashed) in this figure, i.e. \( P_b = Q(\sqrt{E_b}/N_0) \), where \( Q(x) \) is defined as \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \), see e.g. refs. [1], [5].

In Figure 2 it is seen that the performance of the non-coherent receiver in Figure 1, for the same received signal-to-noise ratio \( E_b/N_0 \), is robust with respect to the parameter \( \alpha_0 \). Furthermore, the bit error probability is essentially the same if \( E_b/N_0 \)
Figure 2: The bit error probability \((\log_{10}(P_b))\) for the non-coherent receiver in Figure 1 in the binary case for five different values on the parameter \(\alpha_0\). The dashed line corresponds to coherent ML reception.

is larger than approximatly 10 dB. For smaller values, \(\alpha_0 = 0\) results in a slightly larger error probability than the other four cases. It is interesting to observe that the loss compared with maximum likelihood coherent reception is not that large, and actually less than 1 dB for signal-to-noise ratios larger than 12 dB. It is also interesting to compare the results above with the performance of a non-coherent receiver designed to be ML in the equal energy case, but used in a situation with unequal energies. As an example, consider a situation where one of the two FSK signals is heavily attenuated by the channel. More precisely, let us assume on-off signals \((\alpha_0 = 0)\), and the receiver in Figure 1, designed for the equal energy case \((i.e. \alpha_0 = 1)\). The bit error probability then equals \(P_b = \frac{1 + e^{-2\alpha/2N_0}}{2}\). This example illustrate that the bit error probability can be significantly increased if the non-coherent receiver does not use knowledge about the received signal energies.

3 Frequency Hopping and OFDM-type Methods

One way of introducing diversity in the transmitted signal is to use (block or convolutional) encoding combined with slow (or fast) frequency hopping (FH). As an example, consider a rate 1/2 convolutional encoder combined with \(M\)-ary FSK and slow FH. If \(M = 2\) is used, then each coded bit is transmitted with binary FSK, and if \(M = 4\) then each pair of coded bits is represented by one of the four frequencies. Slow FH can be accomplished by changing the set of \(M\) frequencies e.g. every information bit interval \(T_b\) \((= 1/R_b)\), and it is here assumed that \(F\) different \(M\)-ary FSK signal constellations are used. If the \(M\) parameters \(\alpha_i\) are assumed to be known, then the receiver in Figure 1 can be used (with \(T\) replaced by \(\log_2(M)T_b/2\)) to obtain hard non-coherent \(M\)-ary decisions of the coded bits. These hard decisions are then processed further by the decoding algorithm associated with the encoder. When evaluating the bit error probability for the receiver described above, it is essential to know the error probability properties of the hard decisions. If \(M = 2\), the probability of an error in a hard decision can be evaluated by using expressions (2)-(4) (with \(E_b\) in (4) replaced by \(E_b/2\), since now \(E_b = (1 + \alpha_0)E\)). If, in the binary case, the frequency separation \(f_\Delta\) is chosen to be as small as possible i.e.
\( f_{\Delta} = 2R_b \) Hz, then the bandwidth efficiency for this binary scheme is approximately
\( R_b/(M+1)f_{\Delta} = (6F)^{-1} \) bps/Hz. So, if \( F = 3 \) and \( R_b = 5 \) kbps we need a bandwidth of approximately 90 kHz. For the 4-FSK case above, \( f_{\Delta} = R_b \) can be used which imply a bandwidth efficiency approximately equal to \((5F)^{-1}\) bps/Hz. Consequently, the required bandwidth is in this case approximately 75 kHz if \( F = 3 \) and if the information bit rate is 5 kbps. Note that the bandwidth efficiency for the technique described above is low. Therefore, it is suitable only for low and modest information bit rates \( R_b \).

A more bandwidth efficient way of transmitting data is to use methods which are similar to orthogonal frequency division multiplex (OFDM). The main principle difference between this technique and conventional \( M \)-ary FSK is that more information bits are carried at each frequency. In OFDM, each carrier is normally QAM-modulated, but here we use PAM-modulated carriers. The reason for this is the non-coherent receiver, which uses both quadrature components to explore the inherent diversity in the received signal. Note that non-coherent detection of PAM signals require that they have unequal energies. To illustrate the idea let us investigate an uncoded case. Assume that binary PAM is used at each of the \( M \) orthogonal carrier frequencies. Hence, 1 information bit is carried at each frequency. If the parameters \( \alpha_i \) are assumed to be known, then a modified version of the receiver in Figure 1 can be used to obtain non-coherent binary decisions of the \( M \) information bits. If the \( i \)th binary decision is obtained by comparing \( R_i \) with the threshold \((\sqrt{E_{0,i}} + \sqrt{E_{1,i}})/2\), then the bit error probability can be upper bounded by \( P_e < e^{-\left(\sqrt{E_{1,i}} - \sqrt{E_{0,i}}\right)^2/4N_0} \). Furthermore, if the frequency separation is \( f_{\Delta} = R_b/M \) then the bandwidth efficiency approximately equals \( M/(M+1) \) bps/Hz (without FH). So, if \( M = 10 \) is used in this uncoded case, the required bandwidth is approximately 11 kHz if the information bit rate is 10 kbps.

4 Conclusions
In this paper non-coherent detection has been investigated, and applied to signaling schemes which might be of interest in power line communications. However, much work still remains to find the most suitable method, especially for high information bit rates.

References