A Multi-Path Signal Propagation Model 
for the Power Line Channel in the High Frequency Range
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ABSTRACT
For the use of the mains networks as high speed data path for Internet, voice and data services carrier frequencies within the range from 500 kHz up to 20 MHz must be considered. The development of suitable communication systems and the planning of power line communication networks requires measurement-based models of the transfer characteristics of the mains network in the above-mentioned frequency range.

The heterogeneous structure of the mains network with numerous branches and impedance mismatching causes numerous reflections. Besides multi-path propagation with frequency-selective fading, typical power cables exhibit signal attenuation increasing with length and frequency. The complex transfer function of a power line link can be described by a parametric model in the considered frequency range. Measurements of amplitude and phase response of a sample network with well-known geometry approve the validity of the model. Comparisons with measurements conducted at „live“ mains networks prove the validity of the model also for real network topologies.

1 Introduction
Due to recent demands in the area of communication networks the electrical power supply system is on the way to migrate from a pure energy distribution network to a multi-purpose medium delivering energy, voice and various data services [Zimm98]. Especially Internet access is in the focus of the efforts of various research activities.

The power line network differs considerably in topology, structure and physical properties from conventional telecommunication mediums like twisted pair, coaxial or fibre optic cables. Therefore special communication systems, considering the hostile properties of power line channels, are required [Wald98]. For the design of appropriate communication systems and for planning of power line communication networks, models of the transfer characteristics of the low voltage mains network are required.

Several approaches for modelling the transfer characteristics of power lines can be found in literature. Most of these models represent bottom up approaches describing the behaviour of a network by the components using scattering parameter matrices [Thre91] or four pole impedance and admittance matrices [Barn98], [Dalb97], [Karl97]. These models generally imply detailed knowledge about the components of the network to determine the elements of the matrices. The main drawback of such approaches is the great number of parameters which cannot be determined with sufficient precision. On the contrary to that, a top down approach by regarding the communication channel as a black box and describing its transfer characteristics by a transfer function is the purpose of this paper.

![Figure 1: General channel model](image-url)
derived from channel measurements in contrast to predictive modelling of a network by its geometric dimensions, structure or material properties. In the case of a simple topology (i.e., cable with one branch) the physical reasons for the observed results (cable loss, reflection and transmission factors) can be identified. In the case of more complicated real network topologies the back-tracing of the results to the reasons is often not possible. However, the transfer function may also in such cases be described by the model. The parameters, however, can not be traced back to physical dimensions of the network.

2 Topology of the mains network

In Europe the mains network is typically divided into three sections with different voltage levels. The high voltage, medium voltage and the low voltage section. From a communication point of view not all parts of the mains distribution network are of equal interest. Especially the low voltage distribution grid is of great interest as "last-mile" access network. Hence the model presented in this paper focuses on this section. However, it can also be easily applied to other sections. The communication link from the substation to the backbone network can be implemented by conventional communication links as fibre optics, radio relay links, broadband cables or even by using the medium voltage lines [Zimm98].

The low voltage "local loop access network" between the substation and the customer premises are often operated in a star shaped structure. From a communication point of view they have a similar structure as mobile radio networks consisting of cells and base stations.

In opposite to the telephone copper loop the power line "local loop access network" does not consist of point-to-point connections between substations and customer premises but represents a line bus with the distributor cables and the house service cables. A typical access network link between a substation and a customer (Figure 2) consists of the distributor cable or a series connection of distributor cables with the characteristic impedance $Z_{d}$ and the branching house connection cables with the characteristic impedance $Z_{b,h}$. The house service cable ends at a house connection box. The indoor cabling follows, which is modelled by a termination impedance $Z_{h}(f)$. Each of the transitions at the connections between cables along the propagation path represent changes of impedance and causes reflections.

3 Physical Signal Propagation Effects

3.1 Multi-Path -Signal-Propagation

Due to the structure of the low voltage mains network signal propagation differs from matched lines. Numerous reflections are caused by the joints of the house service cables, house connection boxes and the joints at series connections of cables with different characteristic impedance. Signal propagation does not only take place along a quasi "line of sight" path between the transmitter and the receiver, also additional propagation paths (echoes) must be considered. The result is multi-path signal propagation with frequency selective fading.

![Figure 2: Signal propagation over the power line local loop access network](image)

**Figure 2: Signal propagation over the power line local loop access network**

![Figure 3: Multi-path signal propagation; cable with one tap](image)

**Figure 3: Multi-path signal propagation; cable with one tap**

Multi-path signal propagation is studied at a simple example which can be easily analysed (Figure 3). The link has one branch and consists of the segments (1), (2) and (3) with the lengths $l_1$, $l_2$, $l_3$ and the characteristic impedance $Z_{d1}$, $Z_{d2}$, $Z_{d3}$. 

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Table 1: Signal Propagation paths of the examined sample network

<table>
<thead>
<tr>
<th>Path No.</th>
<th>Way of the signal path</th>
<th>Weighting factor $g_i$</th>
<th>Length of path $d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A \rightarrow B \rightarrow C$</td>
<td>$t_{1B}$</td>
<td>$l_1 + l_2$</td>
</tr>
<tr>
<td>2</td>
<td>$A \rightarrow B \rightarrow D \rightarrow B \rightarrow C$</td>
<td>$t_{1B}t_{3B}t_{1B}$</td>
<td>$l_1 + 2l_3 + l_2$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N</td>
<td>$A \rightarrow B \rightarrow D \rightarrow B \rightarrow C^{N-1}$</td>
<td>$t_{1B}t_{3B}^3t_{1B}$</td>
<td>$l_1 + 2(N-1)l_1 + l_2$</td>
</tr>
</tbody>
</table>

For a simplified consideration $A$ and $C$ are matched, which means $Z_A = Z_{L_1}$ and $Z_C = Z_{L_2}$. The remaining points for reflections are (B) and (C) with the reflection factors

\[ r_{ib} = \frac{a_{ib}Z_{ib} + Z_{ib}}{a_{ib}Z_{ib} + Z_{ib}} \]  
\[ r_{ib} = \frac{Z_0 - Z_{ib}}{Z_0 + Z_{ib}} \]  
\[ r_{ib} = \frac{a_{ib}Z_{ib} + Z_{ib}}{a_{ib}Z_{ib} + Z_{ib}} \]

and the transmission factors

\[ t_{ib} = 1 - |r_{ib}| \]  
\[ t_{ib} = 1 - |r_{ib}| \]

With these assumptions the propagation paths listed in Table 1 are possible. Each path $i$ has a weighting factor $g_i$, representing the product of the reflection and transmission factors along the path. The delay $\tau_i$ of a path

\[ \tau_i = \frac{d_i}{v_p} \]

can be calculated from the length $d_i$ and the phase velocity $v_p$. The losses of real cables cause an attenuation $A(f, d)$ increasing with length and frequency. The signal components of the paths have to be added due to superposition and the transfer function from A to C can be expressed as:

\[ H(f) = \sum_{i=1}^{N} g_i \cdot A(f, d_i) \cdot e^{j2\pi f \tau_i} \]  

All reflection and transmission factors are generally less or equal one.

\[ |r_{ij}| \leq 1, \quad |r_{ij}| \leq 1 \]  
\[ j = 1, 2, 3, \ldots \quad X = A, B, C, D, \ldots \]

Hence the weighting factor $g_i$, a product of transmission and reflection factors, is also less or equal one.

\[ |g_i| \leq 1 \]

The more transitions and reflections occur along a path the smaller the weighting factor $g_i$. Due to the fact that longer paths have higher attenuation they contribute less to the overall signal at the receiving point. It seems reasonable to choose the number of dominant paths $N$ not too large.

Signal propagation in more complicated networks with more branches can be partitioned into paths in a similar way.

### 3.2 Attenuation caused by cable losses

As mentioned above the propagating signals are exposed to attenuation increasing with length and frequency. This section presents a closer look at the losses and derives a mathematical model for them.

\[ U(x) = U_z \cosh(\gamma x) + l_z Z_z \sinh(\gamma x) \]  

Transmission line theory describes the voltage and current along a line (Figure 4) as follows [Stein82]:

\[ U(x) = U_z \cosh(\gamma x) + l_z Z_z \sinh(\gamma x) \]
The Parameters to describe a transmission line are the characteristic impedance $Z_L$ and the propagation constant $\gamma$.

$$Z_L = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$  \hspace{1cm} (12)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$ \hspace{1cm} (13)

Considering a matched transmission line, which is equivalent to regarding only the wave propagating from source to destination, the transfer function of a line of the length $l$ can be expressed as follows:

$$H(f) = \frac{U(x = l)}{U(x = 0)} = e^{-\gamma l} = e^{-\alpha l} e^{j \beta l}$$ \hspace{1cm} (14)

Figure 5 shows a cross section of a typical power line cable with four conductors, widely used in German power distribution systems. When feeding signals into two adjacent conductors, most of the electric field is concentrated between these two conductors. For a first estimate the electric and magnetic field can be approximated by equations describing a micro strip line. The parameters of the cable can be estimated by the geometric dimensions and some material properties.

![Figure 5: Cross section of a typical power line cable (4 conductors)](image)

The inductance per unit length and capacity per unit length can be expressed as follows:

$$C' = \varepsilon_0 \varepsilon_r \frac{r}{a}$$ \hspace{1cm} (15)

$$L' = \mu_0 \mu_r \frac{a}{r}$$ \hspace{1cm} (16)

Considering frequencies in the MHz-range the resistance per unit length is dominated by the skin-effect and can be approximately expressed by

$$R'(f) = \sqrt{\frac{\pi \mu_r}{k r^2 f}} \rightarrow R'(f) \sim \sqrt{f}$$ \hspace{1cm} (17)

following a circular shaped conductor with the diameter $r$. The conductance per unit length

$$G' = 2\pi f C' \tan \delta \rightarrow G' \sim f$$ \hspace{1cm} (18)

is mainly influenced by the dissipation factor of the dielectric material (usually PVC).

The result of using geometry and material properties in the above equations results in $R' \ll \omega L'$ and $G' \ll \omega C'$ in the frequency range of interest. Hence the cables can be regarded as weakly lossy and the characteristic impedance $Z_L$ and the propagation constant $\gamma$ can be determined using the following simplified expressions:

$$Z_L = \sqrt{\frac{L'}{C'}}$$ \hspace{1cm} (19)

$$\gamma = \sqrt{\frac{1}{2Z_L} + \frac{1}{2}} G' Z_L + j \frac{\omega \sqrt{L'C'}}{\text{Re}(\gamma)} = \alpha + j \frac{\text{Im}(\gamma)}{\text{Im}(\gamma)} = \beta$$ \hspace{1cm} (20)

Summarising the characteristic parameters of the cables into the constants $k_1$ and $k_2$ leads to the result

$$\gamma = k_1 \sqrt{f} + k_2 f + j k_3 f.$$ \hspace{1cm} (21)

The real part of the propagation constant, the attenuation loss $\alpha$, increases with frequency. The relation between $\alpha$ and $f$ with a special cable can be proportional to square root of $f$, proportional to $f$ or proportional to a mixture of both, either $k_1$ or $k_2$ dominates.

Based on the derivations starting from physical assumptions and extensive investigations of measured frequency responses the real part of the propagation constant, the cable losses, can be described as

$$\alpha(f) = a_0 + a_1 f^k.$$ \hspace{1cm} (22)

With a suitable selection of the parameters $a_0$, $a_1$ and $k$ the attenuation of a power line cable can be characterised as

$$A(f, d) = e^{-\alpha df} = e^{-\theta_0 + a_1 f^k}.$$ \hspace{1cm} (23)


4 The channel model

4.1 A generalized multi-path-signal propagation model of the transfer function

Combining the multi-path-propagation and the frequency and length depending attenuation finally leads to

\[ H(f) = \sum_{i=1}^{N} \left( g_i \left( \frac{\phi_i(f)}{\alpha_0 + \alpha_1 f^4} \right) e^{-(\alpha_0 + \alpha_1 f^4) d_i} e^{-j2\pi f \tau_i} \right) \]

The run time of a path is described by the delay term. The low pass characteristic, the attenuation increasing with length and frequency, are considered by the attenuation term. The weighting factor \( g_i \) comprises the reflection and transmission factors along a propagation path. Due to the fact that house impedances may exhibit complex, frequency dependent values this weighting factor is for a generalised case chosen complex and frequency dependent. The signal components of the \( N \) paths add together at the receiving point.

4.2 Simplified model

Fortunately, in most practical cases the generalised frequency dependent weighting factors \( g_i \) can be simplified to complex but not frequency depending factors. In heterogeneous real world networks often more than one path with the same delay \( \tau_i \) exists, so that it is very complicated to trace the weighting factors \( g_i \) back to their origins. In such cases the weighting factor simply describes the weight of the path.

The relation between delay \( \tau_i \), length of a path \( d_i \) and phase velocity \( v_p \) is given by

\[ \tau_i = \frac{d_i}{c_0} + \frac{d_i}{v_p} \]

with the speed of light in vacuum \( c_0 \) and the dielectric constant \( \varepsilon \) of the insulating material. This allows the substitute of the delay \( \tau_i \) in (24). The resulting model of the transfer function

\[ H(f) = \sum_{i=1}^{N} g_i e^{-(\alpha_0 + \alpha_1 f^4) d_i} e^{-j2\pi f \frac{d_i}{v_p}} \]

has been widely proved in practice. Table 2 explains the parameters of (26).

| i   | Number of the path. The path with the shortest delay has the index \( i = 1 \) |
| a0, a1 | attenuation parameters |
| k    | exponent of the attenuation factor (usual values between 0.2 and 1) |
| \( g_i \) | Weighting factor for path \( i \), in general complex, can be physically interpreted as the reflection/transmission factors of that path |
| \( d_i \) | length of path \( i \) |
| \( \tau_i \) | delay of path \( i \) |

Equation (26) represents the basis of models describing the complex transfer function of typical power line channels. Using this model all the substantial effects of the transfer characteristics of power line channels in the frequency range from 500kHz to 20 MHz can be modelled by a small set of parameters. Increasing the number of paths \( N \) allows easy control of the precision of the model.

5 Verification of the model by measurements

For verification the results of simulations based on the model (26) are compared with measurements. This was done on the one hand with a sample network with well known topology and geometric dimensions and on the other hand with a real world network.

5.1 Sample network

Figure 6 shows the topology of the sample network. The signal transmitter was located at position A, the receiver at C. A and C were matched with the characteristic impedance of the cable and point D was left open leading to a reflection factor \( r = 1 \).

![Figure 6: Topology of the sample network](image-url)

The transmitted and the received signals were saved in a file using a "Digital Storage Oscilloscope". The complex transfer function and the group delay was
computed off-line (Figure 7). Since the run time of the signal leads to large values in the phase plot, additionally a more detailed “phase detail” plot was generated by subtracting the linear part of the phase. This plot gives a better impression of the frequency ranges with phase distortions.

The reflections at the open tap cause periodical notches in the frequency response, which can easily be seen in the plots (Figure 7). The same sections in the spectrum exhibit phase distortions and changes in the group delay. Because of the non ideal matching at A and C additional small ripples in the frequency response are visible.

Figure 8 shows the results of a simulation of the transfer characteristic with a model based on equation (26) with N=6 paths. The parameter set is listed in Table 3. It is obvious that the simulation and the measurement of the absolute value as well as the phase differs only in some details. The most important conclusion is that the model covers all the essential effects.

Table 3: Parameters of the model of the sample network

<table>
<thead>
<tr>
<th>Path No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length d_i in m</td>
<td>200</td>
<td>221</td>
<td>242</td>
<td>259</td>
<td>266</td>
<td>530</td>
</tr>
<tr>
<td>weighting factor g_i</td>
<td>0.54</td>
<td>0.275</td>
<td>-0.15</td>
<td>0.08</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

k=1 \[ \begin{align*} \alpha_0 &= -2.1 \times 10^{-3} \\ \alpha_1 &= 8.1 \times 10^{-10} \end{align*} \]

5.2 Real world Networks

The model is also applicable to real world networks without detailed knowledge of the network topology. Figure 9 shows the absolute value of the measured frequency response (attenuation) of a power line link with a length of approximately 150 m. The delays of the echoes were determined from the measurement without detailed knowledge of the geometric dimensions of the network. The corresponding lengths were calculated using equation (25) with \( \xi = 4 \).

Table 4: Parameters of the model shown in Figure 9

<table>
<thead>
<tr>
<th>Path No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>delay in ( \mu \text{s} )</td>
<td>1.0</td>
<td>1.25</td>
<td>1.76</td>
<td>2.64</td>
</tr>
<tr>
<td>equivalent length in m</td>
<td>150</td>
<td>188</td>
<td>264</td>
<td>397</td>
</tr>
<tr>
<td>weighting factor ( g_i )</td>
<td>0.4</td>
<td>-0.4</td>
<td>-0.8</td>
<td>-1.5</td>
</tr>
<tr>
<td>( k = 0.5 )</td>
<td>( \alpha_0 = 0 )</td>
<td>( \alpha_1 = 8 \times 10^{-5} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 9 shows the results of a measurement and a simulation with \( N=4 \) paths of that link. The lengths of the paths are listed in Table 4. The plot shows that the essential effects can be described by a relatively simple model. Only at lower frequencies the result of simulation and measurements slightly differ. This is due to the fact that longer echoes, which are not considered in the parameter set, especially influence the lower part of the spectrum.
This example shows the applicability of the model to real world networks. It is feasible to determine a set of parameters for a link and to describe its high frequency signal propagation characteristics without detailed knowledge of dimensions.

6 Summary and conclusions

In this paper a model of the complex transfer function of power line communication links in the frequency range from 500 kHz to 20 MHz has been presented. The model is derived based on physical effects, namely multi-path signal propagation and cable losses. Measurement at a sample network with well known dimensions proved good agreement of the simulation results with measurements. Furthermore the applicability of the model to real world networks was demonstrated.

The presented models offers the possibility to carry out investigations in different network topologies and study their effects on communication systems by the means of simulations. Based on a sufficiently large measurements database signal propagation models for planning power line communication networks can be set up. Besides that reference models of typical channels can be defined for comparison of the performance of different modulation and coding schemes and for future standardisation.

7 References


