Comparison and Optimization of Differentially Encoded Transmission on Fading Channels

Lutz H.-J. Lampe, Robert F.H. Fischer

Laboratorium für Nachrichtentechnik, Universität Erlangen–Nürnberg
Cauerstraße 7/NT
D–91058 Erlangen, Germany
Phone: +49-9131-85-28718, Fax: +49-9131-85-28919
Email: llampe@nt.e-technik.uni-erlangen.de
URL: http://www.nt.e-technik.uni-erlangen.de/~dcg

Abstract — Using orthogonal frequency division multiplexing for power line communication the transmission bandwidth is partitioned into slowly time-varying, flat fading channels. For coding across these channels, several differentially encoded 16-ary signal constellations with incoherent reception are compared with respect to achievable channel capacity. For high bandwidth efficiency the application of 2A8PSK, mixed amplitude/phase shift keying with two concentric rings of 8 signal points each, proves to be advantageous. The amplitude ratio of this constellation is optimized. We show that the performance of 2A8PSK is fairly robust to small variations of the optimum ratio. A comparison of the capacities for 2A8PSK and 16QAM and coherent transmission over the AWGN and the Rayleigh fading channel further motivates the choice of 2A8PSK in the applications under consideration.

1 Introduction

Transmission systems for power line communication have to cope with slowly time-varying, frequency selective channels, cf. e.g. [ADWZ97, Dos97]. By the use of orthogonal frequency division multiplexing (OFDM), e.g. [Bin90, SKJ95], the actual dispersive channel is partitioned into frequency non–selective narrow–band channels. In this contribution, we consider channel coding across these narrow–band channels. Then, the transmission channel is given by a slowly time-varying, frequency non–selective fading channel described by the current channel state (gain) and the phase offset between transmitter and receiver.

In many situations, for transmission over flat fading channels, neither channel state information is available nor reliable coherent reception is possible. For such applications differential encoding at the transmitter and incoherent demodulation at the receiver are convenient. In this paper, we will restrict to conventional differential demodulation, i.e., the demodulation operates on the basis of the last two received symbols. This choice puts the least constraints on the coherence time of the channel and requires much less complexity in practical implementation than larger observation intervals at the receiver (the potential gain by increasing the observation interval is quantified in [CFLM98]).

As mentioned in [Dos97], for power line communication bandwidth efficiency is a subject of growing interest. Hence, in this paper we compare differentially encoded 16–ary constellations, offering spectral efficiencies up to 4 bits/s/Hz, by evaluating the attainable channel capacity.

In conventional M–ary DPSK systems the information is represented in phase changes. For higher spectral efficiencies, mixed differential phase/amplitude modulation ((D)APSK) is attractive, e.g. [Sve95]. We find

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2A8PSK, a signal constellation consisting of points arranged in 2 concentric amplitude rings and 8 uniformly spaced phases, to be a favourite candidate for differentially encoded transmission and high bandwidth efficiency (>2 bit/s/Hz). The optimum value for the amplitude ratio of the two rings is determined. In contrast to previous approaches, e.g. [AS92, Ada96, CNM92], the optimization is done with respect to capacity. The results are thus valid for an optimal receiver structure and powerful channel coding. We show that besides its advantages for differential encoding and incoherent detection 2A8PSK is also well suited for coherent transmission.

2 System Model and Capacity

The system model for differentially encoded transmission and incoherent demodulation is sketched in Figure 1. The discrete-time channel, given in the equivalent low-pass domain [Pro95], comprises the actual power line transmission channel and the transformation due to the IDFT/DFT-pair ((I)DFT: (inverse) discrete Fourier transform) of OFDM. Because of the properties of OFDM, the channel can be assumed to be a stationary, slowly time-varying, frequency non-selective (non-dispersive) Rician fading channel. Usually, channel state and carrier phase offset can be expected to be constant over at least two consecutive symbols.

In order not to require knowledge on the carrier phase and actual channel state, at the transmitter the differential encoder mappes the information to the transitions between two consecutive channel symbols, i.e., the information is carried in amplitude and phase changes, cf. e.g. [DS90].

Signal constellations for the transmit signal \( x \), suited for differential encoding both in phase and amplitude, consist of \( M = \alpha \cdot \beta \) points arranged in \( \alpha \) distinct concentric rings with different radii \( r_i \), \( i = 0, 1, \ldots, \alpha - 1 \), and \( \beta \) uniformly spaced phases. We denote these constellations by \( \alpha\beta\text{PSK} \), which uniquely defines the constellation. As usual for DPSK we restrict the differential symbols \( a \) to be taken from the same signal set as the transmit signal. Then, a simple formulation of the differential encoding results\(^2\). Given a reference symbol (state of the differential encoder) \( s = r_i e^{j\varphi_m} \) and the differential symbol (amplitude and phase increment) \( a = r_j e^{j\varphi_m} \), the encoder outputs \( x = r_{(i+j)} \mod \alpha e^{j(\varphi + \varphi_m)} \). The value of \( x \) then gives the next state of the differential encoder.

At the receiver, two consecutive channel output symbols \( y \) are grouped and comprised into the vector \( y \). Thereby, the blocks are overlapping by one symbol [DS94]. After (perfect) interleaving based on (vector) symbols \( a \) and \( y \), respectively, a memoryless channel is obtained. Hence, the transmission between \( a \) and \( y \) can completely be characterized by a single probability density function (pdf) \( p_Y(y|a) \) of \( y \) for given \( a \) [CFLM98].

Having established the channel model the capacity can be calculated. The capacity \( C \), measured in bit per symbol, is obtained by [Gal68] \(^{2}\) for the moment, interleaving and deinterleaving are ignored.

\[
C := E_{Y,A} \left\{ \log_2 \left( \frac{p_Y(y|a)}{p_Y(y)} \right) \right\}
\]
where $E\{ \cdot \}$ denotes expectation and $p_Y(y)$ is the average pdf of the channel output. Random variables corresponding to signals are denoted by the respective capital letter. As the distribution of $x$ does not depend on the distribution of $a$ no optimization on the channel input distribution can be performed [CFLM98].

3 Numerical Results

In this section numerical results for the capacity over the average signal–to–noise ratio $\bar{E}_s/N_0$ ($\bar{E}_s$: average receive energy per symbol, $N_0$: one–sided noise power spectral density) are presented. In particular, the capacity is evaluated for the additive white Gaussian noise (AWGN) and Rayleigh fading channel, which are the most important special cases of the Rician fading model.

In the case of coherent transmission the capacity of the scalar, actual channel with perfect channel state information (CSI) at the receiver is calculated, cf. e.g. [Gal68].

3.1 Comparison of 16–Point Signal Constellations

Applying 16–ary signal constellations it seems to be advantageous not to solely modulate the phase but also represent information in the amplitude of the transmit signal. Thus, the capacities of D16PSK, D2A8PSK, and D4A4PSK are compared. For D2A8PSK the ring ratio $r := r_1/r_0$ is chosen equal to 2.0 (see Section 3.2). For D4A4PSK the rings are geometrically spaced with ratio $r_{i+1}/r_i = 1.4$, $i = 0, 1, 2$, which was found to be a convenient choice. Note, optimally the ring ratio has to be optimized for each value of the signal–to–noise ratio.

Figure 2 sketches the capacity curves for the AGWN channel and incoherent detection. As can be seen, for all signal–to–noise ratios D4A4PSK is significantly inferior to both D16PSK and D2A8PSK. For the capacity ranging from 2 bit/symbol to 4 bit/symbol, which is interesting in practice, D2A8PSK clearly outperforms D16PSK. Gains up to 3 dB to 4 dB are achievable.

A similar behavior is illustrated in Figure 3 for incoherent detection and Rayleigh fading. The curves correspond to the situations where (a) the receiver does not utilize any CSI and (b) the receiver exploits channel amplitude information (CAI), i.e. knowledge of the fading amplitude, is available at the receiver side, respectively. Without CSI, due to the amplitude modulation, the difference in capacity of D2A8PSK and D16PSK decreases for capacities $> 2$ bit/symbol compared to the AWGN channel case. But only D2A8PSK (and D4A4PSK) considerably benefits from CAI at the receiver. As expected, CAI provides only marginal capacity gains in the case of pure phase modulation.

3.2 Optimization of D2A8PSK

For best performance the ring ratio $r$ of the 2A8PSK constellation has to be optimized. Since Euclidean distance is not a significant parameter in the case of differential demodulation the optimum $r$ is not given simply by the geometry of the constellation.

The capacities for several values of $r$ are plotted in Figure 4 for the AWGN channel and the Rayleigh fading channel without CSI, respectively. Here, we focus on the capacity range corresponding to incoherent transmission with high bandwidth efficiency ($> 2$ bit/s/Hz). According to the curves the ring ratio equal to 2.0
seems to be an advantageous choice for both channel models. Only for certain values of the signal-to-noise ratio marginally higher capacities are achieved on the AWGN channel with ring ratios equal to 1.8 and 2.2, respectively. Noteworthy, variations of \( r \) within 1.8 and 2.2 does not lead to a significant loss in performance.

### 3.3 Coherent Transmission with 2A8PSK

In the previous sections, 2A8PSK has been found to be an appropriate signal constellation for differentially encoded transmission and we have optimized the ring ratio for conventional differential detection. In order to further assess the 2A8PSK signal constellation, we compare the capacities of the AWGN and Rayleigh fading channel for coherent transmission using 2A8PSK and conventional 16QAM. Here, no differential encoding is performed at the transmitter and CSI is available at the receiver side.

Figure 5 shows the respective capacity curves. The ring ratio of 2A8PSK is again \( r = 2.0 \). Judging 2A8PSK by its normalized minimum squared Euclidean distance, we would expect 2A8PSK to be inferior to 16QAM. But, astonishingly, the capacity of 2A8PSK is almost the same as or even exceeds the capacity of 16QAM. The result is similar in spirit to the findings in [WFH], where it is shown that even for the AWGN channel Euclidean distance is not the most important parameter in the context of near-capacity channel coding.

It should be noted that the optimization of the ring ratio yields very similar results as for D2A8PSK. For capacities between 2 bit/symbol and 4 bit/symbol \( r = 2.0 \) is the favourite choice. Again, the capacity turns out to be fairly robust to variations of the \( r \) within 1.8 and 2.2.

### 4 Conclusions

In this paper power and bandwidth efficient differentially encoded transmission over fading channels is studied. Differential encoding of amplitude and phase of the transmitted signal is used. As an appropriate measure of performance channel capacity is regarded.

After establishing the system model for differential encoding and incoherent detection over a Rician fading channel the capacities for several 16-ary signal constellations are calculated as a function of the signal-to-noise ratio. For a fixed ring ratio D2A8PSK proves to be superior for highly bandwidth efficient transmission.

The optimization of the ring ratio based on the capacity has been performed. A value of 2.0 appears to be optimum in the relevant capacity range. Small variations of this ring ratio does not significantly change the capacity of D2A8PSK.

The comparison of the capacities for 2A8PSK and 16QAM and coherent transmission over the AWGN and the Rayleigh fading channel shows that besides the advantages of 2A8PSK for differential encoding and incoherent detection the constellation is a convenient choice in general.

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References


Figure 2: Capacities of D16PSK (solid line), D2A8PSK (dashed line), D4A4PSK (dash-dotted line). AWGN channel.

Figure 3: Capacities of D16PSK (solid line), D2A8PSK (dashed line), D4A4PSK (dash-dotted line). Rayleigh fading channel. Squares: without CSI, Stars: with CAI at receiver side.
Figure 4: Capacities of D2A8PSK with different ring ratios. AWGN and Rayleigh fading channel.

Figure 5: Capacities of 2A8PSK (solid line) and 16QAM (dashed line). AWGN and Rayleigh fading channel with CSI at the receiver.