DMT-FDMA as a multiple access technique for wideband upstream powerline communications

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Abstract

This paper investigates the use of DMT-FDMA in the context of wideband upstream powerline communications. The multiple access component relies on a share of the different carriers among the users. The issue of reception under ideal synchronisation assumptions and a simplified noise environment is addressed. Fractional FIR linear and decision feedback joint detectors are derived. A tone allocation method is proposed, that intends to maximize the total bit rate under a fairness constraint between the users. The performance is compared with the classical guard time equalization technique. Keywords: DMT, OFDM, Multiple Access, Joint Detection.

1 Introduction

The use of the low voltage power distribution network as an access to high speed (i.e. several Mb/s) communications services is considered today as a possible alternative to other existing access systems such as cable modems on the CATV network and DSL modems on the copper wire telephone network. The part of the network under consideration in the present paper is that located between the medium voltage/low voltage transformer and the meter in the home. We focus on the upstream transmission system, i.e. from the end users to the optical network access supposed to be located at the cable headend, next to the transformer.

This access network has to be shared among different users possibly requesting different bit rates: thus, a multiple access modulation scheme has to be selected. That scheme has to be well suited to the high frequency-selectivity of the users channels, due to the propagation over non-ideal cables in addition to the multiple reflections of the signals at cable transitions and unmatched cable terminations. There is also a lot of non white background noise coupling onto the cable. Finally the transmission may also be corrupted by impulsive noise, but the purpose of this analysis is only to take into account the effect of stationary noise sources.

The DMT scheme [1] can be directly extended to the multiuser configuration thanks to its multicarrier nature [2]. Each user will be provided with a given subset of carriers, in a so-called ‘DMT-FDMA’ technique. In the present paper we propose a DMT-FDMA scheme for multiuser transmission over a powerline network. As an alternative to the classical guard time equalization technique, we investigate the use of linear and decision-feedback joint detectors. We compare both schemes in terms of performance and complexity. Another issue is the distribution of the tones to the users depending on their respective channels.

2 The channel model

International standards and restrictions concerning the use of high bandwidths in powerline cables for high speed communication purposes are under development. The available bandwidth will be down-limited as the lower part of the spectrum is dedicated to the existing low speed services and the power distribution

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Low voltage channel from LT to NT; (t | t ≤ 5) — unused taps are left open

Figure 1: Powerline channels transmittance

itself. We assume a free bandwidth $B_\text{a}$ of several MHz starting from a lower limit $f_{\text{min}}$ arbitrarily fixed to 1 MHz.

In wired communications [3], the restriction on the emitted power usually consists of a transmission mask giving the upper limits of the signal power spectral density $\gamma_\text{m}(\omega)$. We assume a flat mask $\gamma_{\text{max}}$ of -60 dBm/Hz along the bandwidth $B_\text{a}$ (the same limit as in VDSL, a wired system with a similar bandwidth).

Powerline networks can vary in a wide range, depending on the number of users, the actual topology, the cables nature, the line termination, the noise environment, etc. We chose a typical network model with $K_\text{u} = 5$ user-modems (Network Terminations, NT$_1$ to NT$_5$) located along the main distribution cable at increasing distances from the Line Termination (LT). End-to-end channel responses were computed using the model proposed in [4]. Multiple reflections of the signals at cables transitions and on ill-terminated cables derivations give rise to frequency-dispersive channel transmittances, as illustrated by figure 1. Powerful equalization will be needed to counteract these effects. Both NT’s and LT will be provided with an analog front end consisting of at least a line-driver for tuning the signal to the right power level, a high-pass service-splitter and a low-pass filter designed to reduce out-of-band signal components.

We consider the case of an uplink, namely transmission between NT’s and the LT. The problem of duplexing will not be considered in this paper. The signal $x_k(t)$ of user $k \in [1, K_\text{a}]$ is transmitted over an analog channel with impulse response $c_k(t)$ comprising both the channel itself, the line driver and the transmit analog filters. The signal received at the LT is given by

$$r_\text{a}(t) = \sum_{k=1}^{K_\text{u}} x_k(t) \odot c_k(t) + n_\text{a}(t)$$

where $\odot$ denotes the convolution operator.

In this paper, we restrict ourselves to the case of an additive white gaussian noise $n_\text{a}(t)$ with two-sided power spectral density $\gamma(\omega) = N_0/2$ equal to -134 dBm/Hz. It has to be underlined that more complex additive noise configurations should be taken into account in realistic situations, including colored and non-stationary noises. This simplified configuration, however, is convenient for the study of multiple access techniques.
3 The DMT-FDMA transmission scheme

DMT signals synthesis relies on the separation of the emitted complex symbols into \( N_p/2 - 1 \) parallel streams (called tones) at rate \( T_b \) through the use of an order-\( N_p \) Inverse Fast Fourier Transform operator (IFFT). The DC and Nyquist components are supposed to be left unused. In the set of \( N_p/2 - 1 \) tones, only \( K_p \) ones are available as the lower ones correspond to the unused part of the spectrum. This set of \( K_p \) tones is divided among the \( K \) users in subsets \( P_k \) thanks to a tone allocation algorithm in the LT. The signal \( x_k(t) \) transmitted by user \( k \) is given by

\[
x_k(t) = \sum_{l=-\infty}^{\infty} \sum_{p \in P_k} I_p(l) \Phi_p(t - lT_b) + \sum_{l=-\infty}^{\infty} \sum_{p \in \mathcal{P}_k} I'_p(l) \Phi'_{p}(t - lT_b) \tag{2}
\]

where

\[
\Phi_p(t) = \frac{1}{\sqrt{N_p}} \sum_{n=-\nu}^{N_p-1} e^{2\pi j n t_p} g(t - nT_p) \tag{3}
\]

and \( g(t) \) is an interpolation filter. The second term in (2) is necessary to make the signal real.

Defining

\[
\Phi_p(t) = \Phi_p(t) + j\Phi'_p(t) \tag{4}
\]

\[
I_p = I_p^0 + jI'_p \tag{5}
\]

we have the equivalent real expression for the DMT signal:

\[
x_k(t) = 2 \sum_{l=-\infty}^{\infty} \sum_{p \in P_k} I_p^0(l) \Phi^0_p(t - lT_b) - 2 \sum_{l=-\infty}^{\infty} \sum_{p \in \mathcal{P}_k} I'_p(l) \Phi'_p(t - lT_b) \tag{6}
\]

\( T_p = T_b/(N + \nu) \) denotes the nominal DMT sample duration. An optional cyclic prefix of length \( \nu \) can be added to the signal in order to provide an easier equalization, the so-called 'guard-time' equalization technique, which is specific to the DMT basis functions. As the sample duration \( T_p \) is fixed to 0.5/(\( f_{\text{min}} + B_a \)), the use of the cyclic prefix adds a penalty of \( N_p/(N_p + \nu) \) to the baud rate \( 1/T_b \) and the resulting capacity. The whole scheme is summarized in figure 2.

4 The DMT-FDMA receiver

The conventional detection mechanism in the DMT scheme implies the use of a direct Fast Fourier Transform (FFT) operator, after removal of the cyclic prefix. Single coefficient adjustments are sufficient at each FFT output to compensate for the channel effect and produce estimates of the transmitted QAM symbols.

If we make use of the guard time technique, it would be helpful to use a time domain equalizer (TEQ) [5] in order to shorten the main energy of the channel impulse response down to the cyclic prefix duration. In the multiuser case, however, different subcarriers are affected by different channels and the TEQ method is not directly applicable. If we want to keep the guard time duration below a reasonable value or if we don’t make use of the TEQ technique, the use of an equalizer in the frequency domain (FEQ) will be needed, which implies a high complexity.

We propose to give up the cyclic prefix operation and the use of the FFT at the receiver side. As an alternative, a joint detector (JD) can be directly placed at the output of the sampling device and produce optimal estimates of the transmitted symbols according to a MMSE criterion as explained in the next section.

As the transmit filter is not ideal, transmitted signals are not strictly bandlimited to 0.5/\( T_p \). It is possible to gain some information about the transmitted symbols by sampling the received signal at a rate \( 1/T_s = M_p/T_p \) with \( M_p > 1 \).
With $f(t)$ the receiver filter, we obtain a fractional rate description of the received signal:

$$r_n(t) \otimes f(t) = \sum_{p \in P_n} \sum_{l=-\infty}^{\infty} I_p^l(t) h_p^l(t-T_l) + \sum_{k=1}^{K_n} \sum_{l=-\infty}^{\infty} I_p^l(t) h_p^l(t-T_l) + n_f(t)$$

(7)

where $h_p^l(t) = 2\Phi_p^l(t) \otimes c_k(t) \otimes f(t)$ and $h_p^l(t) = -2\Phi_p^l(t) \otimes c_k(t) \otimes f(t)$. We define $r(l) = r_n(lT_b) \otimes f(lT_b)$. Using the polyphase notation, we define $M_p N_p$ real polyphase components $r_{k_1}(n)$ ($k_1 \in [0, M_p(N_p - 1)]$) as follows:

$$r_{k_1}(n) = r(nT_b + k_1T_b)$$

(8)

Assuming that the polyphase components of the channel impulse responses are of length $L_1 + L_2 + 1 = L + 1$ and using a matrix formalism, we can write:

$$\mathbf{g}(n) = \begin{bmatrix} \mathbf{H}(L_2) & \cdots & \mathbf{H}(0) & \cdots & \mathbf{H}(-L_1) \end{bmatrix} \begin{bmatrix} I(n-L_2) \\ \vdots \\ I(n) \\ \vdots \\ I(n+L_1) \end{bmatrix} + \mathbf{u}(n)$$

(9)

where $\mathbf{g}(n)$ and $\mathbf{u}(n)$ are vectors of $M_p N_p$ polyphase components, $\mathbf{H}(l)$ with $l \in [-L_1, L_2]$ are matrices of size $M_p N_p \times 2K_p$ and $I(n)$ is the vector of $2K_p$ real symbols transmitted by the users at time $nT_b$. It should be understood that this equation is a matrix formulation of a continuous transmission process, not a block based transmission scheme. From this representation, we will investigate FIR linear and decision-feedback receiver structures.
5 FIR joint detection

The FIR linear joint detector will build estimates of the $2K_p$ symbols streams from the $M_p N_p$ received samples streams in the following way:

$$
\hat{I}(n) = \begin{bmatrix}
\mathcal{C}(K_2) & \cdots & \mathcal{C}(0) & \cdots & \mathcal{C}(-K_1)
\end{bmatrix}
\begin{bmatrix}
\tilde{z}(n - K_2) \\
\vdots \\
\tilde{z}(n) \\
\vdots \\
\tilde{z}(n + K_1)
\end{bmatrix}
$$

In a DF receiver, one also uses the decisions on a number $K_3$ of previous blocks of symbols. Furthermore, we assume that each time a new symbol is detected in the current block, it is used for the next estimation to be performed. Hence, the estimation is computed as follows (perfect past decisions are assumed):

$$
\hat{I}(n) = \mathcal{C}^{K_1,K_2} I_4(n) - \mathcal{D}(K_3) \begin{bmatrix}
I(n - K_3) \\
\vdots \\
I(n)
\end{bmatrix}
$$

(10)

where $\mathcal{D}(0)$ is a lower triangular matrix with 0's on the main diagonal as only past decisions are available for the feedback section. Forward and feedback sections of the JD are calculated with a MMSE criterion as stated in [7].

6 Bit rates estimation

Each user is provided with a number of pairs of parallel data streams consisting of PAM symbols at a baud rate $1/T_b$. The constellation size of a specific stream can be adapted to the reception quality. Considering a target symbol error probability $P_s$, and assuming that the decision variable after JD is approximately gaussian, the maximum number of bits per PAM symbol is given by the well-known formula [1]:

$$
b = \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNIR}}{\Gamma} \right)
$$

(12)

where $\Gamma$, the 'SNR-gap', is a function of $P_s$, and the SNIR is defined as the 'Signal to Noise plus Interference ratio' on the symbols stream. Interference is in fact residual interference coming from adjacent symbols on the same tone (Inter Symbol Interference), from symbols on the other tones of the same user (Inter Carrier Interference), and from symbols from other users (Multiple Access Interference).

The total system bit rate is given by the following expression:

$$
C_{DMT} = \gamma_{\text{duplex}} \frac{N_p}{N_p + \nu} \left( f_{\text{min}} + B_a \right) \frac{K_p}{N_p/2} \log_2 \left( \prod_{m=1}^{2K_p} \frac{\text{SNIR}_p}{\Gamma} \right)^{\frac{1}{2K_p}}
$$

(13)

$\gamma_{\text{duplex}}$ is introduced to take into account the duplexing scheme, supposed here to be Time Division Duplexing (TDD). It is the ratio between the time dedicated to the uplink transmission and the total time for both transmissions (uplink and downlink).

A useful bound on the attainable equalizer performance is the 'Matched Filter Bound' (MFB) defined as the best reachable SNR in the absence of interfering streams, when a single symbol is sent on the useful stream. This best SNR is given by $2E_s/N_0$, where $E_s$ is the PAM symbol energy at the receiver.

With non-ideal channels, all the tones won’t have the same potentialities for a given user. Considering the $K_a$ different channels, an important problem to solve is the partitioning of the tones set in order to maximize the resulting bit rate.
7 Tone allocation

Assuming an initial set of $K_p$ tones to be shared among the $K_u$ users, there are $(K_u + 1)^{K_p}$ possible allocations if we consider the possibility of leaving some tones unused. The choice of the optimal allocation is a problem of best assignment that can be solved by a long and systematic search. A difficulty comes from the use of equalizers to obtain the final SNIR that will decide for the bit rate. Indeed, the equalizers coefficients depend on the chosen resource allocation, and recomputation of the equalizer would be needed for every considered allocation.

To overcome this problem, we recall that the MFB is a good indicator of the reachable bit rate for a specific resource on a specific channel, especially if the joint detection process is efficient. We propose to select the allocation of the available resources on that simplified criterion. The basis for that work is just the computation of $K_u$ size-$K_p$ vectors giving the metrics $C_{k,p} = \frac{1}{2K_p} \left[ \log_2(1 + \frac{2C_i(k,p)}{K_p}) + \log_2(1 + \frac{2C_i(k,p)}{N_0T}) \right]$ with $k \in [1, K_u]$ and $p \in [1, K_p]$.

To select the best allocation, we still need a criterion of optimality. A possible choice would be the maximization of the total system bit rate, but the corresponding optimum could be the allocation of all resources to the user with the best channel, which is of course not acceptable. Some constraints have to be defined that guarantee a fair share between the users. In real systems, these would depend on the effective needs of the users. As an example, we assume all users request an access to the same type of service. The optimisation criterion will be the total bit rate $C_{tot}$ under a fairness constraint on the 'users imbalance index' $\Delta_{users}$ defined as:

$$\Delta_{users} = \frac{\sigma^2_C}{\bar{C}} = \frac{1}{K_u} \sum_k \left( \frac{C_k - \bar{C}}{\bar{C}} \right)^2 \leq \delta$$

where $\bar{C} = \frac{1}{K_u} \sum_k C_k = \frac{1}{K_u} C_{tot}$ is the average user bit rate and $\delta$ is a small tolerance coefficient.

It is assumed that allocation schemes satisfying (14) exist. This will be true if the imbalance between the channels is not too large and the number of available tones is high enough to compensate for that imbalance by the allocation of a different amount of tones to the users. In [6], a strict procedure is proposed for a similar problem, based on Kuhn-Tucker conditions. A heuristic algorithm can also be used to compute the best allocation as proposed in [7], with similar performance.

The method proposed here is not optimal in the sense that it is based on the MFB. As the effective SNIR obtained at the output of the detector can be quite smaller, a new iterative procedure must follow that initial allocation scheme in order to reach condition (14) on the user bit rates.

8 Complexity

It can be shown very easily that the complexity of the different detectors (expressed in number of real multiplications per second) is given by the following expressions:

$$C_z^{\text{Linear-JD}} = \frac{2(B_u + f_{\text{min}})}{N_p} \left[ 2K_p M_p N_p (K_1 + K_2 + 1) \right]$$

$$C_z^{\text{DF-JD}} = \frac{2(B_u + f_{\text{min}})}{N_p} \left[ 2K_p M_p N_p (K_1 + K_2 + 1) + \left( (2K_p - 1)^2 / 2 + 4K_p^2 K_3 \right) \right]$$

$$C_z^{\text{FFT-CP}} = \frac{2(B_u + f_{\text{min}})}{N_p + \nu} \left[ 2N_p \log_2(N_p) + 4K_p \right]$$

It appears that joint detection requires a much higher complexity, which could be acceptable as only one JD is needed at the LT for the whole PLC network.

9 Results and conclusions

Figure 3 illustrates the power spectral density of the signal emitted by NT3. The available bandwidth goes from 1 to 11 MHz ($B_u = 10 MHz$). The effective spectral occupation is slightly higher because of
the filter rolloff. A length-128 IFFT was used, without cyclic prefix ($N_p = 128, \nu = 0$). 58 out of the 64 tones were available for the transmission ($K_p = 58$). After the tone allocation algorithm, user-2 was provided with 9 tones. Figure 4 gives the SNIR profiles at the output of the detectors, together with the MFB. The lower figure gives the tone allocation scheme. The global system bit rate here is summarized in table 1, with equalizers sizes $K_1 = K_2 = K_3 = 4$ (reasonable values as $L_1$ and $L_2$ are in the same range), $\gamma_{\text{duplex}}$ left to 1 and a symbol error probability $P_e = 10^{-7}$. Please also remember the poor noise model used here.

The joint detector could be replaced by a lower complexity detector consisting of the combination of a very large FFT and a sufficient guard time. Corresponding bit rates are illustrated on figure 5, for a system working with a size-4096 FFT, as a function of the cyclic prefix length $\nu$. It appears that the best performance (85 Mbits/s) is below that of the DF-JD. Furthermore, accurate users synchronisation is necessary to get such a low multiple access interference level. Another drawback is the higher system latency involved by the large FFT-size. The main advantage of this solution, of course, is the much lower complexity as compared with the joint detector.

**References**


Figure 3: Power Spectral Density of DMT-FDMA transmitted signals - $N_p = 128$, $K_p = 58$, $\nu = 0$

Figure 4: SNIR on received tones (DMT-FDMA) - $M_p = 2$, $K_1 = K_2 = K_3 = 4$

Figure 5: Bit rates for a size-4096 DMT-FDMA system with guard time equalization vs cyclic prefix length