Medium Access Scheme for TDMA

Lutz H.–J. Lampe[1]

Lehrstuhl für Nachrichtentechnik II LNT, Universität Erlangen–Nürnberg
Cauerstraße 7/NT, D–91058 Erlangen, Germany

Phone: +49-9131-85-28718, Fax: +49-9131-85-28919, Email: LLampe@LNT.de

Abstract — **Robust medium access signaling for power–line networks using time–division multiple access is discussed. We propose coding, signal design, and receiver processing which allow to detect signaling users from the superimposed received signal, while exploiting full diversity of the frequency–selective power–line channel. While the presented scheme is applicable to any modulation technique, it is particularly suited to orthogonal frequency division multiplexing.**

Index terms: medium access, dispersive channel, OFDM, superimposed codes, multi–access communication

1 Introduction

We consider the situation where N users communicate over a power–line network. Multiple access is accomplished by time–division multiplexing, i.e. time–division multiple access (TDMA), where all subscribers are synchronized to a TDMA framing structure provided by a master station. Within the TDMA scheme distinct time slots are reserved for signaling *transmission requests* of users to the master station, i.e., for the *medium access* of subscribers. As transmission–request signals do not carry any payload, their frequency and their duration should be kept as low as possible. Conversely, to ensure quality of service, transmission requests have to be processed with high probability and short delay by the master. These requirements prohibit the assignment of different signaling slots to each subscriber, which in turn implies that *collisions* of transmission requests are possible. Here, a collision is defined as simultaneous signaling of a transmission request by different users. Of course, collisions should either be avoided, or the master station should be able to *resolve* collisions, i.e., to recognize transmission requests of the individual users from the superimposed received signal.

After introducing the system model in Section 2, in Section 3 we discuss the design of transmit sequences and signal processing of the received signal for robust medium access signaling, exploiting full frequency diversity and allowing wide collision resolvability. To increase the maximum number N of users in the power–line network, the effective combination of signaling based on orthogonal sequences with superimposed coding is shown in Section 4. Although the demand of complete collision resolvability has to be dropped, collisions of any $1 < m < N$ users can be resolved. Since for the detection of transmission requests threshold decisions are involved, Section 5 provides an outline of the optimization of threshold values. Finally, conclusions are drawn in Section 6.

2 System Model

Let the vector $\boldsymbol{\xi}_n = [\xi_{1,n}, \ldots, \xi_{L_s,n}]^T$ ($\cdot^T$: transposition) represent the discrete–time version of the equivalent complex baseband transmission–request signal of user n, $1 \leq n \leq N$. In general, its L_s components are complex numbers $\xi_{k,n} \in \mathbf{C}$, $1 \leq k \leq L_s$. Assuming the frequency–selective power–line transmission channel to be time–invariant during one TDMA slot, the equivalent discrete–time channel impulse response for user n is denoted by

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$h_n = [h_{1,n}, \ldots, h_{L_h,n}]^T$, where L_h is the maximum expected length of h_n, $1 \le n \le N$. It is reasonable to assume the channel state, i.e., h_n, to be unknown at the receiver side.

To avoid interference between signals of different TDMA slots, a guard period of $L_h - 1$ samples is required. Here, we choose the guard symbols to constitute a cyclic extension of the signal ξ_n, which is discarded at the receiver. Then, the linear convolution of transmit signal and channel impulse response is converted into a cyclic convolution, which allows for simple channel equalization as used in orthogonal frequency division multiplexing (OFDM) [1] and, equivalently, which is suited to low–complexity channel estimation in frequency domain [2], cf. also [3]. Accordingly, the received signal $r = [r_1, \ldots, r_{L_s}]^T$ reads [4]

$$r = \sum_{n \in \mathcal{N}} H_n \xi_n + n \,, \tag{1}$$

where $\mathcal{N}$ denotes the set users sending a transmission request in the same time slot, H_n is the $L_s \times L_s$ cyclic matrix with the first column $[h_n^T, 0, \ldots, 0]^T$, and $n = [n_1, \ldots, n_{L_s}]^T$ represents mutually independent additive white Gaussian noise (AWGN) with variance σ_n^2 per complex component. Defining Ξ_n as $L_s \times L_h$ matrix containing in the k^{th} column the vector ξ_n cyclically shifted by $k-1$ samples, and interchanging the roles of signal and channel, (1) transforms into

$$r = \sum_{n \in \mathcal{N}} \Xi_n h_n + n \,. \tag{2}$$

Of course, to reduce the probability of collisions, the signaling time slot can be divided into subslots, and a user chooses one subslot for signaling transmission requests at random. This can be interpreted as slotted Aloha (TDMA) [5] within one TDMA frame. However, besides the fact that collisions are still possible even if only two users are sending simultaneously, this approach is rather inefficient as the portion of guard time increases.

Next, regarding the application of OFDM, the frequency–domain signals $x_n \triangleq 1/\sqrt{L_s} \cdot \text{DFT}_{L_s \times L_s} \cdot \xi_n$ ($\text{DFT}_{L_s \times L_s}$ denotes the $L_s \times L_s$ discrete Fourier transform (DFT) matrix) can be designed such that different OFDM subcarriers are assigned to different users. Since the OFDM subcarriers are orthogonal by definition, collisions can be resolved at the receiver by applying a DFT to r. Unfortunately, this frequency–division multiplexing (FDMA) approach is clearly not feasible for transmission over *frequency–selective* channels, i.e., for power–line communications.

From the discussion above it seems to be natural to regard the signal ξ_n as code or signature sequence of user n, i.e., to think of *code–division multiple access (CDMA)*. For robust medium access full frequency diversity should be exploited, i.e., the transmit signal ξ_n should be *white*, which in turn implies that the frequency–domain signal x_n allocates all OFDM subcarriers with equal power.

3 Detection and Signature Sequences

Regarding (1), the problem of medium access signaling is to detect from r whether ξ_n is present or not. Since the channel state information, i.e., the entries of H_n, is not available at the receiver, but the signature sequence ξ_n is known *a priori*, the detection problem is conveniently reformulated as *channel estimation* problem (cf. (2)). In particular, as ξ_n is required to be white, all L_h channel coefficients $h_{k,n}$, $1 \le k \le L_h$, are to be estimated from r. Since the set $\mathcal{N}$ of active users is not known, but to be determined, in its most general form (2) has to be rewritten as[2]

$$r = \Xi \cdot h + n \,, \tag{3}$$

where the definitions $\Xi \triangleq [\Xi_1, \ldots, \Xi_N]$ and $h \triangleq [h_1^T, \ldots, h_N^T]^T$ are applied. Clearly, for users n not contributing to r, i.e., $n \notin \mathcal{N}$, $h_{k,n} = 0$, $1 \le k \le L_h$, is true. Denoting the estimated version of h_n by $\hat{h}_n$ and using

[2] Of course, users having already established a connection can be excluded from the set $\mathcal{N}$.

signal energy as appropriate measure, which indeed corresponds to maximum ratio combining, decision variables ($\cdot^H$: Hermitian transposition)

$$d_n = \hat{\boldsymbol{h}}_n^H \cdot \hat{\boldsymbol{h}}_n \tag{4}$$

are obtained. Now, a simple decision rule is as follows: if $d_n > t$ ($d_n \leq t$), user n has (not) sent a transmission request. Here, $t \in \mathbb{R}^+$ is a conveniently chosen threshold accounting for the disturbance by noise $\boldsymbol{n}$.

Equation (3) models a multiuser communication scenario, where each user occupies L_h dimensions in an L_s dimensional signal space. Optimum estimates $\hat{\boldsymbol{h}}_n$ are obtained if Ξ is a unitary matrix, i.e., $\Xi^H \Xi = \boldsymbol{I}_{N \cdot L_h \times N \cdot L_h}$ ($\boldsymbol{I}_{N \cdot L_h \times N \cdot L_h}$ denotes the $N \cdot L_h \times N \cdot L_h$ identity matrix). Then, the matched filter receiver [6] yields

$$\hat{\boldsymbol{h}} = \Xi^H \cdot \boldsymbol{r} = \boldsymbol{h} + \tilde{\boldsymbol{n}} \,, \tag{5}$$

with $\hat{\boldsymbol{h}} \triangleq [\hat{\boldsymbol{h}}_1^T, \ldots, \hat{\boldsymbol{h}}_N^T]^T$ and AWGN $\tilde{\boldsymbol{n}}$ with variance σ_n^2 per complex component. We observe that signature sequences $\boldsymbol{\xi}_n$ of different users are orthogonal and hence collisions are resolvable at the receiver. Of course, the maximum number N of users is restricted to

$$N \leq L_s / L_h \,. \tag{6}$$

If larger values N are demanded, the situation resembles an overloaded multiuser system, which requires multiuser detection techniques, cf. e.g. [6]. However, since the components of $\hat{\boldsymbol{h}}_n$ are generally continuous-valued, performance degradation compared to orthogonal signaling is unavoidable. We will show in Section 4 how N can be increased over (6) while still guaranteeing resolvability of collisions of any $1 < m < N$ users.

For unitary Ξ the conditions ($\delta[\cdot]$ denotes the Kronecker delta)

$$\begin{aligned} [\xi_{1,n}, \ldots, \xi_{L_s,n}] \cdot [\xi_{L_s-k+1,\nu}, \ldots, \xi_{L_s,\nu}, \xi_{1,\nu}, \ldots, \xi_{L_s-k,\nu}]^H &= \delta[n-\nu] \cdot \delta[k] \,, \\ & 1 \leq n, \nu \leq N \,,\; 0 \leq k \leq L_h - 1 \end{aligned} \tag{7}$$

have to be met. A solution to (7) is found by choosing $\boldsymbol{\xi}_n$ as the by $(n-1) \cdot L_h$ coefficients cyclically shifted version of the sequence $\boldsymbol{\xi}_1$ with $\boldsymbol{\xi}_1$ having a *perfectly periodic autocorrelation function*, i.e., $\boldsymbol{\xi}_1$ is orthogonal to all its $L_s - 1$ cyclically shifted versions [7]:

$$\boldsymbol{\xi}_n = [\xi_{L_s-(n-1)L_h+1,1}, \ldots, \xi_{L_s,1}, \xi_{1,1}, \ldots \xi_{L_s-(n-1)L_h,1}]^T \,, \quad 2 \leq n \leq N \,, \tag{8}$$

and

$$[\xi_{1,1}, \ldots, \xi_{L_s,1}] \cdot [\xi_{L_s-k+1,1}, \ldots, \xi_{L_s,1}, \xi_{1,1}, \ldots \xi_{L_s-k,1}]^H = \delta[k] \,, \quad 0 \leq k \leq L_s - 1 \,. \tag{9}$$

Sequences $\boldsymbol{\xi}_1$ with the desired property (9) can be found in e.g. [7], cf. also [3, 8]. Of particular interest are sequences whose coefficients $\xi_{k,1}$ stem from an M–ary phase–shift keying (PSK) signal constellation [3, 7, 8]. Of course, due to (9) the signals $\boldsymbol{\xi}_n$ are white in frequency domain.

If, as for OFDM, signals are specified in frequency domain rather than in time domain, (8) and (9) translate into

$$\boldsymbol{x}_n = \mathrm{diag}\left\{1, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{L_s}L_h}, \ldots, \mathrm{e}^{-\mathrm{j}\frac{2\pi}{L_s}L_h(L_s-1)}\right\}^{n-1} \cdot \boldsymbol{x}_1 \,, \quad 2 \leq n \leq N \,, \tag{10}$$

and

$$\boldsymbol{x}_1 \cdot \boldsymbol{x}_1^H = \mathrm{diag}\{1, 1, \ldots, 1\} \,, \tag{11}$$

respectively, where $\mathrm{diag}\{z_1, \ldots, z_{L_s}\}$ is the $L_s \times L_s$ diagonal matrix with elements z_k, $1 \leq k \leq L_s$. Now, the frequency–domain coefficients $x_{k,n}$ can be chosen as M–ary PSK symbols.

Finally, we note that the identical construction of frequency–domain signals is advocated in [9] to obtain optimum training sequences for channel estimation in OFDM systems with multiple transmit antennas.

4 Increasing Number of Users for Medium Access Signaling

As already noted in the previous section, requiring complete collision resolvability combined with exploiting the entire frequency diversity, the number of users within the network is limited by (6). To overcome this limit, we have to drop the demand of all possible collisions to be resolvable. Since the probability of simultaneous transmission–request signaling of $U \leq N$ users can be assumed to decrease considerably with increasing U, it is reasonable to require collisions of only up to $U_{\max}$ signals, e.g., $U_{\max} = 2, 3$, to be resolvable at the receiver. If more than $U_{\max}$ subscribers try to send a transmission request during the same time slot, the resulting collision should still be detectable.

As outlined in the following, this goal is achieved by applying binary *superimposed codes* [10] to the present situation. More specifically, each user n, $1 \leq n \leq N$, is assigned to several, say W, signature sequences $\xi_{\ell_w(n)}$, $1 \leq \ell_w(n) \leq L \leq L_s/L_h$, $1 \leq w \leq W$, which in turn implies that the same sequence is assigned to several users. The numbers $\ell_w(n)$ are the positions of the non–zero entries of the n^{th} word $c_n = [c_{1,n}, \ldots, c_{L,n}]^T$ of a binary superimposed code of N words with length $L \leq L_s/L_h$. Now, each user n is identified by its code word c_n, $1 \leq n \leq N$, rather than by its (single) signature sequence as assumed in the previous sections. To maintain an invariance of the system performance with respect to the considered user n, and for the sake of simplicity we assume W to be fixed for all users, which corresponds to superimposed codes of constant weight W.

4.1 Equivalent OR–channel

The rationale behind the use of superimposed codes is that the overall transmission of medium access signals is equivalent to the transmission over a logical OR–channel, cf. e.g. [11, 12]. Following the exposition in Sections 2 and 3 the received signal now reads

$$\begin{aligned} r &= \sum_{n=1}^{N} H_n \sum_{w=1}^{W} \xi_{\ell_w(n)} + n \\ &= [\Xi_1, \ldots, \Xi_L] \cdot \left[\sum_{n,w:\ell_w(n)=1} h_n^T, \sum_{n,w:\ell_w(n)=2} h_n^T, \ldots, \sum_{n,w:\ell_w(n)=L} h_n^T \right]^T + n \, . \end{aligned} \tag{12}$$

Accordingly, the matched filter output is given by

$$\hat{h} = \left[\hat{h}_1^T, \hat{h}_2^T, \ldots, \hat{h}_L^T\right]^T = \left[\sum_{n,w:\ell_w(n)=1} h_n^T, \sum_{n,w:\ell_w(n)=2} h_n^T, \ldots, \sum_{n,w:\ell_w(n)=L} h_n^T \right]^T + \tilde{n} \, . \tag{13}$$

Clearly, the decision variables from (4) $d_l = \hat{h}_l^H \hat{h}_l$, $1 \leq l \leq L$, do not represent an estimate of the energy of one, but of the sum of channel impulse responses corresponding to users n with $\ell_w(n) = l$, $1 \leq w \leq W$. If one or more out of all users n with $\ell_w(n) = l$ sends a transmission request, d_l is expected to exceed the threshold t. Consequently, the binary decision $\hat{c}_l$ with $\hat{c}_l = 1$ if $d_l > t$, $\hat{c}_l = 0$ if $d_l \leq t$, is the output of the digit–by–digit Boolean sum of the binary code words c_n assigned to the transmitting users $n \in \mathcal{N}$:

$$\hat{c} = [\hat{c}_1, \ldots, \hat{c}_L]^T = \bigvee_{n \in \mathcal{N}} c_n \, , \tag{14}$$

which is the equivalent OR–channel of our medium access signaling scheme.

4.2 Application of Superimposed Codes

The distinct property of superimposed codes is that, to a prescribed extent, from digit–by–digit Boolean sums of code words the code words forming the sum can be deduced. More specifically, zero–false–drop codes of order m (ZFD$_m$) introduced in [10] ensure that no sum of up to m code words includes any other code word not used in this sum. Exploiting this property for our situation, i.e., choosing $\boldsymbol{c}_n$ as words of an ZFD$_m$ code, from $\hat{\boldsymbol{c}}$ all users having sent a transmission request can be detected as long as not more than m users are transmitting simultaneously. Moreover, if more than m users are transmitting simultaneously, an irresolvable collision is detected.

Denoting by $\lambda_{\max}$ the maximum number of digit positions in which any two code words have ones, the value of m is bounded by [10] (floor$\{z\}$ denotes the integer part of $z \in \mathbb{R}^+$)

$$m \geq \text{floor}\{(W-1)/\lambda_{\max}\} . \tag{15}$$

Regarding constant–weight W codes with minimum Hamming distance $d = 2(W - \lambda_{\max})$, (15) gives

$$m \geq \text{floor}\{(W-1)/(W-d/2)\} . \tag{16}$$

For given code length L, which is limited by L_s/L_h, and desired w, constant–weight codes with a maximum number of words, which gives the number N of users, have to be found. For this, the extensive tables of constant–weight codes in [13] provide a useful reference.

Table 1 shows possible parameter sets for $L \leq L_s/L_h$, code weight W, and maximum number of users N, if $m = 2$ is sufficient. We observe that for e.g. $L = 23$ applying superimposed codes the number of users can be increased to $N = 253$ (compared to $N = 23$ without code).

4.3 Design Example

We illustrate the above considerations for a specific example.

Let the transmit signal without cyclic extension consist of $L_s = 256$ samples, and assume the channel impulse responses to have not more than $L_h = 12$ taps. If signature sequences are favorably specified in time domain, their coefficients can be taken from a 16PSK constellation based on Frank–sequences [7]:

$$\xi_{k,1} = \mathrm{e}^{\mathrm{j}\frac{2\pi}{16}((k-1) \bmod 16)\text{floor}\{(k-1)/16\}} , \quad 1 \leq k \leq 256 . \tag{17}$$

On the other hand, when using OFDM, the frequency–domain coefficients may stem from a 64PSK constellation, because $x_{k,l}$, $1 \leq l \leq L$, is obtained from rotating $x_{k,1}$ ($|x_{k,1}| = 1$) by a multiple of $2\pi/64$ (see Eq. (11)).

We further assume that simultaneous signaling of transmission requests of more than two users is much less likely than signaling of only one or two users. Hence, $m = 2$ is appropriate, i.e., collisions caused by any two users are resolvable, but collisions of more than two signals are detected as irresolvable. According to Table 1 and $L \leq 256/12 \approx 21.3$, a maximum of $N = 120$ users is allowed to participate in the communication network. Each user is assigned to $W = 7$ signature sequences $\boldsymbol{\xi}_{\ell_w(n)}$ and $\boldsymbol{x}_{\ell_w(n)}$, respectively.

Table 1: *Possible parameter sets L, W, N for $m = 2$ based on constant–weight codes taken from [13].*

$L \leq L_s/L_h$	9	13	14	15	16	17	19	20	21	22	23
code weight W	3	3	5	5	5	5	5	5	7	7	7
# N of users	12	26	28	42	48	68	76	84	120	176	253

5 Optimization of Threshold Values

So far, we have assumed that the threshold t is ideally chosen, i.e., if the decision variable d_l, $1 \leq l \leq L$, contains noise only, $d_l \leq t$ is true, and $d_l > t$ holds if one user n with $\ell_w(n) = l$ for some $1 \leq w \leq W$ has been sending. However, since different transmission characteristics apply to different users, e.g., regarding users close to or remote from the master station in a power-line network, an absolute value of t is not appropriate. Instead, a sliding threshold depending on the distribution of d_l over l seems to be advantageous. More specifically, rather than comparing d_l with t, ratios d_l/d_k, $1 \leq l, k \leq L$, are compared to t.

Furthermore, using ZDF_m codes a very limited number of hypotheses has to be tested only. These tests are favorably processed in a number of successive binary hypothesis tests applying varying thresholds. Assuming slowly time-varying channel characteristics and the average signal-to-noise (SNR) required for reliable data transmission to be in a certain range $\mathrm{SNR}_{\mathrm{min}} \leq \mathrm{SNR} \leq \mathrm{SNR}_{\mathrm{max}}$, these thresholds can be optimized based on analytical expressions which allow to estimate *false alarm* and *detection probabilities* [14]. Using such an approach, for design examples as in Section 4.3 the probability of detecting a user which is not present (false alarm) or not detecting a present user (missed detection) is about 10^{-10} for optimized threshold values [15].

6 Conclusion

The issue of medium access signaling for power-line networks with time-division multiple access is discussed. Using a cyclic signal extension to account for the frequency-selective channel, we show how orthogonal signaling can be combined with superimposed codes such that interfering transmission-request signals can be resolved at the receiver to a certain extent. Depending on the design goals, a flexible trade-off between maximum number of users and capability to resolve collisions is obtained. For robust signaling over the frequency-selective power-line channel, the proposed scheme exploits full frequency diversity.

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